Lesson 15

July 12, 2016

1. Plot the point given in rectangular coordinates by $(3, 4, 10)$, then find the cylindrical coordinates for the point.
2. Plot the point given in cylindrical coordinates by $(5, \frac{5\pi}{6}, 4)$, then find the rectangular coordinates for the point.
3. Write the equation $3x^2 + y^2 = z$ in cylindrical coordinates.
4. Sketch the solid given by the inequalities $0 \leq z \leq 4 - r$ and $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$.
5. Evaluate $\int_{\frac{\pi}{2}}^{\pi} \int_{1}^{2} \int_{0}^{2-r} r \, dz \, dr \, d\theta$. What does this integral tell us?
6. Set up the following integrals using cylindrical or rectangular coordinates.
   (a) $\int\int\int_E dV$, where E is the solid above the cone $z^2 = x^2 + y^2$ and below the sphere $x^2 + y^2 + z^2 = 8$
   (b) $\int\int\int_E xy \, dV$, where E is the solid inside both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$
   (c) $\int\int\int_E f(x, y, z) \, dV$, where E is the solid below $x + y + 2z = 2$ in the first octant
   (d) $\int\int\int_E f(x, y, z) \, dV$, where E is the solid below $x^2 + y^2 + 2z = 2$ in the first octant
   (e) $\int\int\int_E yz \, dV$, where E is the solid bounded above by $z = x + y$, below by the xy-plane, and by $y = x^2$ and $x = y^2$

Answers:

1. $(5, \tan^{-1}(\frac{4}{3}), 10)$
2. $(\frac{-5\sqrt{3}}{2}, \frac{5}{2}, 4)$
3. $z = 3r^2\cos^2\theta + r^2\sin^2\theta$ or $z = 2r^2\cos^2\theta + r^2$
4. The part of a cone opening downward from $(0, 0, 4)$ (rectangular coordinates) whose projection onto the xy-plane is the part of the circle $x^2 + y^2 = 2$ in the second and third quadrants.
5. $\frac{\pi}{3}$. This gives the volume of the part of the cone $z = 2 - \sqrt{x^2 - y^2}$ above the second quadrant, lying inside $x^2 + y^2 = 2$ and outside $x^2 + y^2 = 1$.
6. (a) cylindrical; $\displaystyle\int_0^{2\pi}\int_r^{\sqrt{8-r^2}}\int_0^4 r\,dz\,dr\,d\theta$

   (b) cylindrical; $\displaystyle\int_0^{2\pi}\int_0^1\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r^3\sin\theta\cos\theta\,dz\,dr\,d\theta$

   (c) rectangular; $\displaystyle\int_0^2\int_0^{2-x}\int_0^{1-0.5x-0.5y} f(x,y,z)\,dz\,dy\,dx$

   (d) cylindrical; $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^{\sqrt{2}}\int_0^{1-0.5r^2} f(r\cos\theta, r\sin\theta, z)r\,dz\,dr\,d\theta$

   (e) rectangular; $\displaystyle\int_0^1\int_{x^2}^{\sqrt{x}}\int_0^{x+y} dz\,dy\,dx$